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The mass of the remaining part is

$$M_1=2\int_0^1\int_0^{2\pi}\int_{\cos^{-1}(1/a)}^{\frac{1}{2}\pi}r^4\sin\theta drd\varphi d\theta=\frac{4\pi}{5a}.$$

But $M=M_1$.

$\therefore \frac{4\pi}{5a}=\frac{2\pi}{5a}(a^6-a^5-a+1)$.

$\therefore a^6-a^5-a-1=0$.

$\therefore a^3-(1/a^3)=a^2+(1/a^2)$, or $t^3-t^2+3t-2=0$ where $t=a-(1/a)$.

Let $t=s+\frac{1}{3}$, $\therefore s^3+\frac{8}{3}s=\frac{29}{27}$.

$\therefore s=.381891$, $t=.715224$.

$a=\frac{1}{2}[t+\sqrt{(4+t^2)}]=1.4196$ inches nearly.

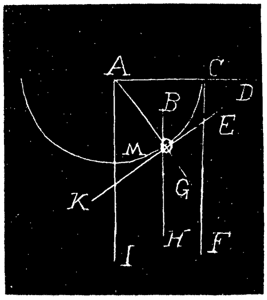
Similarly solved by *J. SCHEFFER*, his answer being 3.7462 inches.

72. Proposed by REV. A. L. GRIDLEY, Pastor of First Congregational Church, Kidder, Mo.

Prove that the motion of a ball falling through the earth influenced by gravity alone would be similar to the motion of a pendulum.

I. Solution by the PROPOSER.

It is an established fact that if a body should fall through an opening through the center of the earth, the net force drawing it toward the center is as the remaining distance to be traversed.



The pendulum ball follows the same law. It is evident that when the rod is horizontal all the force of gravity is exerted upon it to move it along its vertical tangent. When the rod is vertical, there is no net available force to move it from its position.

Making it general, by resolution of forces, the net force of gravity available to move the ball along the tangent EK is, in the figure, equal to the $\measuredangle AMB$. But this angle$=\measuredangle IAM$, as lines AI and BH are parallel and are cut by line AG.. But $\measuredangle IAG$ represents the remaining distance for the ball to pass through. Thus the law in both cases is the same.

Inference. 1st. To vibrate in the same time the whole arc—semi-circle—of the pendulum must equal the diameter of the earth.

2d. Vibrating past the center for any distance would require the same time as from surface to surface.

3d. The net available force acting is in inverse proportion to the distance passed through.

Similarly solved by *ALOIS F. KOVARIK*.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; CHARLES C. CROSS, Libertytown, Md.; CHARLES E. MEYERS, Canton, Ohio, and J. SCHEFFER, A. M., Hagerstown, Md.

The equation of motion is $d^2x/dt^2=-gx/R$. Where g=gravity=$32\frac{1}{6}$ feet, R=3963 miles=radius of earth. Multiplying by $2dx$ and integrating, we get $(dx/dt)^2=-gx^2/R+C$.

Let $x=a$ when $dx/dt=0$. Then $C=ga^2/R$.

$\therefore (dx/dt)^2=(g/r)(a^2-x^2)\ldots\ldots(1)$.

$\therefore dt=\sqrt{(R/g)}.dx/\sqrt{(a^2-x^2)}$.

$\therefore t=\sqrt{(R/g)}\cos^{-1}(x/a)+C_1$.

Since $t=0$ when $x=a$, $C_1=0$.

$\therefore t=\sqrt{(R/g)}\cos^{-1}(x/a)\ldots\ldots(2)$.

Equation (2) represents simple harmonic motion. From (1), velocity is zero when $x=\pm a$ and a maximum when $x=0$.

When $x=0$, $t=\frac{1}{2}\pi\sqrt{(R/g)}$. This time is the same for any value of a between 0 and R.

$2t$=time of complete vibration$=\pi\sqrt{(R/g)}$.

$2t$=42 minutes, 12 seconds.

Also solved by *J. H. DRUMMOND*.

DIOPHANTINE ANALYSIS.

69. Proposed by JOSIAH H. DRUMMOND, LL. D., Counselor at Law, Portland, Me.

Two right angled triangles have the same base which is a mean proportional between the two perpendiculars: find a general solution, that will give integral values for all the sides of both triangles.

I. Solution by the PROPOSER.

Let x=the base, y=the perpendicular of one triangle, and z=that of the other triangle. Then $x^2=yz\ldots\ldots(1)$, and $x^2+y^2=\square\ldots\ldots(2)$, and $x^2+z^2=\square$ $\ldots\ldots(3)$. Substituting yz for x^2 in (2) and (3), we have $zy+y^2=\square\ldots\ldots(4)$, and $zy+z^2=\square\ldots\ldots(5)$. Take $y=mz$ and substituting in (4) and (5), and reducing, we have $m+1=\square=$, say, p^2; and $m=p^2-1$.

$m(m+1)=\square\ldots\ldots(6)$, substituting the value of m in (6), we have $p^2(p^2-1)=\square$. Hence we have to make $p^2-1=\square=$, say, $(q-p)^2$. Hence $p=(q^2+1)/2q$ and $m=[(q^2-1)/2q]^2$; and $y=z[(q^2-1)/2q]^2$. $x=\sqrt{(yz)}=z[(q^2-1)/2q]$; to obtain integral values take $z=4q^2$, and we have $x=2q(q^2-1)$, $z=4q^2$ and hypotenuse$=2q(q^2+1)$, and $x=2q(q^2-1)$, $y=(q^2-1)^2$ and hypotenuse$=(q^2+1)(q^2-1)$, in which q may be any number >1.

If the value of q is an odd number, it is manifest that one fourth of each of the foregoing values will be integral, and we shall have smaller values.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let p, h be the perpendiculars; c, d the hypotenuses; k the common base.